Strategy for Solving Linear Equations

1.	***Eliminate*** all ***grouping symbols*** using the Distributive Property.
2.	***Clear all fractions/decimals*** if needed and combine like terms on each side.
3.	***Gather the variables*** to one side. Remember that the side we choose does not matter because if a = b then b = a.
4.	***Isolate the variable quantity.*** Gather all of the constants on the opposite side of the variable quantity.
5.	***Solve*** for the variable. Obtain the goal of $1 \cdot x = ?$.
6.	***Check*** the solution.

Solve: $4(2x-3)+7=3x+5$

$4(2x-3)+7=3x+5$	***Eliminate*** **parentheses using the Distributive Property.**
$8x-12+7=3x+5$	
$8x-5=3x+5$	**Combine like terms.**
$8x-3x-5=3x-3x+5$	***Gather*** **variables to the left-hand side.**
$5x-5=5$	
$5x-5+5=5+5$	***Isolate*** **the variable quantity on the left-hand side.**
$5x=10$	
$\frac{5x}{5}=\frac{10}{5}$	***Solve*** **for the variable by dividing both sides by 5.**
$1 \cdot x=2$	**Achieve goal: 1· x = ?**

Make sure to *check* to see if you answer is correct by placing your solution into every variable in the original equation and make sure that both sides of the equation are equal to each other.

Solve: $\frac{x}{4} + \frac{x}{3} = \frac{-x}{6} + 3$

$$\frac{x}{4} + \frac{x}{3} = \frac{-x}{6} + 3$$

Find the LCD of this equation.

$$24\left(\frac{x}{4} + \frac{x}{3}\right) = 24\left(\frac{-x}{6} + 3\right)$$

Clear fractions **by multiplying both sides by LCD = 24.**

$$\frac{24}{1}\left(\frac{x}{4}\right) + \frac{24}{1}\left(\frac{x}{3}\right) = \frac{24}{1}\left(\frac{-x}{6}\right) + 24(3)$$

Distribute the LCD

$$\frac{24x}{4} + \frac{24x}{3} = \frac{-24x}{6} + 72$$

$$6x + 8x = -4x + 72$$

Simplify to eliminate the fractions.

$$14x = -4x + 72$$

Combine like terms.

$$14x + 4x = -4x + 4x + 72$$

Gather **variables to the left-hand side.**

$$18x = 72$$

$$\frac{18x}{18} = \frac{72}{18}$$

Solve **for the variable by dividing both sides by 18.**

$$x = 4$$

Achieve goal: $1 \cdot x = ?$

Make sure to *check* to see if you answer is correct by placing your solution into every variable in the original equation and make sure that both sides of the equation are equal to each other.

Solve: $\frac{x-3}{3}+\frac{x+2}{4}=\frac{11}{6}$

$$\frac{x-3}{3}+\frac{x+2}{4}=\frac{11}{6}$$ Find the LCD of this equation.

$$12\left(\frac{x-3}{3}+\frac{x+2}{4}\right)=12\left(\frac{11}{6}\right)$$ *Clear fractions* by multiplying both sides by LCD = 12.

$$\frac{12}{1}\left(\frac{x-3}{3}\right)+\frac{12}{1}\left(\frac{x+2}{4}\right)=\frac{12}{1}\left(\frac{11}{6}\right)$$ Distribute the LCD.

$$\frac{12x-36}{3}+\frac{12x+24}{4}=\frac{132}{6}$$

$$4x-12+3x+6=22$$ Simplify to eliminate the fractions.

$$7x-6=22$$ Combine like terms.

$$7x-6+6=22+6$$ *Isolate* the variable quantity on the left-hand side.

$$7x=28$$

$$\frac{7x}{7}=\frac{28}{7}$$ *Solve* for the variable by dividing both sides by 7.

$$x=4$$ Achieve goal: $1 \cdot x = ?$

Make sure to *check* to see if you answer is correct by placing your solution into every variable in the original equation and make sure that both sides of the equation are equal to each other.

Solve: $0.25x + 0.10(x - 3) = 0.05(22)$

$$0.25x + 0.10(x - 3) = 0.05(22)$$

$$100[0.25]x + 100[0.10](x - 3) = 100[0.05](22)$$

Clear **the decimals by multiplying both sides by 100.**

$$25x + 10(x - 3) = 5(22)$$

Eliminate **parentheses using the Distributive Property.**

$$25x + 10x - 30 = 110$$

$$35x - 30 = 110$$

Combine like terms.

$$35x - 30 + 30 = 110 + 30$$

Isolate **the variable quantity on the left-hand side.**

$$35x = 140$$

$$\frac{35x}{35} = \frac{140}{35}$$

Solve **for the variable by dividing both sides by 7.**

$$x = 4$$

Achieve goal: 1 · x = ?

Make sure to *check* to see if you answer is correct by placing your solution into every variable in the original equation and make sure that both sides of the equation are equal to each other.

All the linear equations that we have solved thus far have had a single solution. An equation with a single solution is called a *conditional equation.* Not all linear equations have a single solution, however. Some have none at all.

Inconsistent

An equation that has no solution is said to be *inconsistent.* The solution set of any inconsistent equation is the empty set.

Ex.

$$
\begin{aligned}
2(x+3) &= 2x+7 \\
2x+6 &= 2x+7 \\
2x-2x+6 &= 2x-2x+7 \\
6 &= 7 \quad \text{False}
\end{aligned}
$$

Solution set: { } or Ø

Because this last equation is always false, there is no real number that, when substituted for *x*, will cause the original equation to hold true. There is no solution to this equation, so it is inconsistent. Some equations hold true for all numbers.

Identity

An equation that has all real numbers as its solution is called an *identity.* The solution set to an identity is the set of real numbers.

Ex.

$$
\begin{aligned}
7x-6(x+1) &= -3-(3-x) \\
7x-6x-6 &= -3-(3-x) \\
x-6 &= -3-3+x \\
x-6 &= -6+x \\
x-x-6 &= -6+x-x \\
-6 &= -6 \quad \text{True}
\end{aligned}
$$

Solution set: $\{x \mid x \in \mathbb{R}\}$

Because this last equation is always true, every real number that is substituted for *x* will cause the original equation to hold true. It is an identity.